

Modelling Market Power Cost in the Assessment of Transmission Investment Policies

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Abstract—This paper develops a mathematical tool for modelling market power cost in transmission expansion planning decisions. The mathematical modelling is based on the game theory in applied mathematics and the concept of social welfare in microeconomics. We assume the generating companies as Cournot players and the Transmission System Operators as a regulated social transmission planner. To tackle the multiple Nash equilibria problem, the concept of worst-Nash equilibrium is defined and mathematically formulated. The developed mathematical structure is a mixed-integer linear programming problem. This closed form mathematical structure can be solved efficiently using the available computational packages.

Index Terms—Transmission Augmentation, Market Power, worst-Nash equilibrium

I. INTRODUCTION

TRANSMISSION planning is complex, involving consideration of the impact of a transmission augmentation under a large number of future demand and supply scenarios. The transmission planning problem is well understood in the context of a vertically-integrated electricity industry,[1]. In principle, if the liberalised electricity market is sufficiently competitive, the same tools and techniques that have been developed for transmission planning in the context of an integrated electricity industry can be applied. However, two new issues arise: (a) The first issue is the problem of generator market power, (b) The second is coordination between generation and transmission investment. The focus of this paper is on the first issue.

Modelling market power cost in the process of transmission expansion planning is an issue which is not researched well. It is shown that additional transmission capacity can reduce the market power cost and improve competition between rival generating companies, [2]. Modelling of the market power cost in the process of transmission expansion planning is complex and it is not researched well. In [3], [4], [5] no literature is referred about the modelling of market power for economic transmission augmentation.

Reference [6] suggests two heuristic procedures for transmission augmentation. The authors use unconstrained oligopoly equilibrium for the set of producers' bids while bids from the demand side are assumed to be fixed and derived

from analysis of the existing market data.

References [7] and [8] show that generators benefit from a reduction in transmission capacity. Using a simplified version of the power network in California, [9] has quantified the impact of local market power and transmission capacity.

The TEAM methodology introduced by the California ISO [10] is a good model for analysing economic-efficiency-based transmission augmentation. However, the California ISO method is an ad-hoc and heuristic method.

To the authors' best knowledge, all approaches proposed for modelling market power cost in the process of are either ad hoc or lack an efficient solution technique.

This paper derives a mixed-integer linear programming problem for assessment of transmission investment policies taking into account the market power cost. The derived mathematical structure is closed-form and it can be solved efficiently. In section II, the strategic generating companies, the worst-Nash equilibrium, and the transmission system operator are mathematically modelled. Section III includes a case study and section IV concludes this paper.

II. MATHEMATICAL FORMULATION

A. Strategic Generating Companies

A strategic generating company is modelled using the static leader-follower game. In this game, the leader is the generating company and the follower is the electricity market operator. The electricity market operator runs a bid-based security-constrained economic dispatch. The generating company calculates its profit using the endogenous prices and the generation quantities available from the dispatch process.

This bilevel programming problem is nonlinear and nonconvex and consequently it is very hard to be solved efficiently. This paper uses the Karush-Kuhn-Tucker optimality conditions and the disjunctive nature of complementary slackness conditions, and it converts this bilevel programming problem into a mixed-integer linear programming problem. The final mixed-integer linear programming problem is given in (1-1) and (1-2).

The bid-based security-constrained economic dispatch is formulated in (1-1). The objective of this optimisation problem is the total operating cost. The first constraint is the energy balance equation. Second and third constraints represent the generic transmission constraints. The generation

capacity of the generating units is modelled in the fourth and fifth constraints. Finally, the last constraint models the system reference node.

$$\begin{aligned}
 & \text{Max} - \sum_g \sum_n G_{g,n} MC_g GQTY_g \\
 \text{s.t.} \\
 & \sum_n B_{m,n} t_n = \left(\sum_g G_{g,m} GQTY_g - \sum_d D_{d,m} DQTY_d \right) \forall m \longleftrightarrow u \\
 & - \sum_n H_{l,n} t_n \geq -TCAP_l \forall l \longleftrightarrow v^{t \max} \\
 & \sum_n H_{l,n} t_n \geq -TCAP_l \forall l \longleftrightarrow v^{t \min} \\
 & -GQTY_g \geq -GCAP_g \forall g \longleftrightarrow v^{g \max} \\
 & GQTY_g \geq 0 \forall g \longleftrightarrow v^{g \min} \\
 & t_N = 0 \longleftrightarrow w
 \end{aligned} \tag{1}$$

Where in (1), $G_{g,n}$ is the matrix of connection points of generating units, MC_g is the short-run marginal cost of generating unit g , $GQTY_g$ is the economically dispatched generation quantity, $B_{m,n}$ is the acceptance matrix, t_n is the node angle, $D_{d,m}$ is the matrix of connection points of demand units, $DQTY_d$ is the demand level, $H_{l,n}$ is the matrix transmission lines topologies, $TCAP$ is the transmission lines capacities, $GCAP_g$ is the capacity of generating units, and $u, v^{t \max}, v^{t \min}, v^{g \max}, v^{g \min}, w$ are the Lagrange multipliers of their associated constraints. A profit-maximising generating company can be formulated as in (2).

$$\begin{aligned}
 & \text{Max} \pi_p = \sum_{g \in P} (u_g - MC_g) GQTY_g \\
 \text{s.t.} \\
 & 0 \leq GCAP_g \leq GMAX_g \\
 & \text{Max} - \sum_g \sum_n G_{g,n} MC_g GQTY_g \\
 \text{s.t.} \\
 & \sum_n B_{m,n} t_n = \left(\sum_g G_{g,m} GQTY_g - \sum_d D_{d,m} DQTY_d \right) \forall m \tag{2} \\
 & - \sum_n H_{l,n} t_n \geq -TCAP_l \forall l \\
 & \sum_n H_{l,n} t_n \geq -TCAP_l \forall l \\
 & -GQTY_g \geq -GCAP_g \forall g \\
 & -GQTY_g \geq 0 \forall g \\
 & t_N = 0
 \end{aligned}$$

The Karush-Kuhn-Tucker optimality conditions for optimisation problem (1) (in addition to the feasibility conditions) are as follows;

Dual Feasibility:

$$\begin{aligned}
 & - \sum_n G_{g,n} MC_g + \sum_n u_n G_{g,n} - v_g^{g \max} + v_g^{g \min} = 0 \forall g \\
 & - \sum_m u_m B_{m,n} - \sum_l v_l^{t \max} H_{l,n} + \sum_l v_l^{t \min} H_{l,n} = 0 \forall n \\
 & v_l^{t \max}, v_l^{t \min} \geq 0 \forall l \\
 & v_g^{g \max}, v_g^{g \min} \geq 0 \forall g
 \end{aligned} \tag{3}$$

Complementary Slackness Conditions:

$$\begin{aligned}
 & v_l^{t \max} \left(TCAP_l - \sum_n H_{l,n} t_n \right) = 0 \forall l \\
 & v_l^{t \min} \left(TCAP_l + \sum_n H_{l,n} t_n \right) = 0 \forall l \\
 & v_g^{g \max} (GMAX_g - GQTY_g) = 0 \forall g \\
 & v_g^{g \min} GQTY_g = 0 \forall g
 \end{aligned} \tag{4}$$

The dual feasibility constraints are all linear. The complementary slackness conditions can be linearised by introducing a binary variable. The expression $XY = 0$ can be written as $Xb + Y(1-b) = 0$ where b is a binary variable. The last expression can be written as two linear equations: $-M(1-b) \leq X \leq M(1-b)$ and $-Mb \leq Y \leq Mb$. This linearisation method is used to linearise all complementary slackness conditions.

By substituting the inner optimisation problem in (2) by its equivalent linearised KKT conditions, all constraints in optimisation problem (2) are now linear. The only non-linear term is the objective function π_p .

The strong duality condition holds for the inner optimization problem.

$$\begin{aligned}
 & \sum_g v_g^{g \max} GMAX_g - \sum_n \left(\sum_d D_{d,n} DQTY_d \right) u_n + \sum_l v_l^{t \max} TCAP_l + \\
 & \sum_l v_l^{t \min} TCAP_l = - \sum_g \sum_n G_{g,n} MC_g GQTY_g \Rightarrow \\
 & \sum_g v_g^{g \max} GMAX_g = \sum_n \left(\sum_d D_{d,n} DQTY_d \right) u_n - \sum_l v_l^{t \max} TCAP_l + \\
 & - \sum_l v_l^{t \min} TCAP_l + \sum_g \sum_n G_{g,n} MC_g GQTY_g - \sum_{g \notin P} v_g^{g \max} GMAX_g
 \end{aligned} \tag{5}$$

The right-hand side of equation (5) involves only linear terms. Now we show that the left-hand side is the profit function of portfolio p.

$$\begin{aligned}
& - \sum_n G_{g,n} MC_g + \sum_n u_n G_{g,n} - v_g^{g \max} + v_g^{g \min} = 0 \Rightarrow \\
& - \sum_{g \in P} \sum_n G_{g,n} MC_g QTY_g + \sum_{g \in P} \sum_n u_n G_{g,n} QTY_g \\
& - \sum_{g \in P} v_g^{g \max} QTY_g + \sum_{g \in P} v_g^{g \min} QTY_g = 0 \Rightarrow \\
& \pi_p = \sum_{g \in P} v_g^{g \max} GMAX_g
\end{aligned} \tag{6}$$

Substituting (5) in (6) yields the mixed-integer linear programming problem in (7).

$$\begin{aligned}
Max \pi_p = & \left(\begin{array}{l} \sum_n \left(\sum_d D_{d,n} DQTY_d \right) u_n \\ - \sum_l v_l^{t \max} TCAP_l - \sum_l v_l^{t \min} TCAP_l \\ + \sum_{g \in GenCo_p} \sum_n G_{g,n} MC_g QTY_g - \sum_{g \in GenCo_p} v_g^{g \max} GCAP_g \end{array} \right) \\
s.t. & 0 \leq GCAP_g \leq GMAX_g \\
& \sum_n B_{m,n} t_n = \left(\sum_g G_{g,m} QTY_g - \sum_d D_{d,m} DQTY_d \right) \forall m \\
& -TCAP_l \leq \sum_n H_{l,n} t_n \leq TCAP_l \forall l \\
& 0 \leq QTY_g \leq GCAP_g \forall g \\
& - \sum_n G_{g,n} MC_g + \sum_n u_n G_{g,n} - v_g^{g \max} + v_g^{g \min} = 0 \forall g \\
& - \sum_m u_m B_{m,n} - \sum_l v_l^{t \max} H_{l,n} + \sum_l v_l^{t \min} H_{l,n} = 0 \forall n \\
& - \hat{A}b_l^{t \max} \leq TCAP_l - \sum_n H_{l,n} t_n \leq \hat{A}b_l^{t \max} \forall l \\
& - \hat{A}b_l^{t \max} \leq v_l^{t \max} \leq \hat{A}b_l^{t \max} \forall l \\
& - \hat{A}b_l^{t \min} \leq TCAP_l + \sum_n H_{l,n} t_n \leq \hat{A}b_l^{t \min} \forall l \\
& - \hat{A}b_l^{t \min} \leq v_l^{t \min} \leq \hat{A}b_l^{t \min} \forall l \\
& - \hat{A}b_g^{g \max} \leq GCAP_g - QTY_g \leq \hat{A}b_g^{g \max} \forall g \\
& - \hat{A}b_g^{g \max} \leq v_g^{g \max} \leq \hat{A}b_g^{g \max} \forall g \\
& - \hat{A}b_g^{g \min} \leq QTY_g \leq \hat{A}b_g^{g \min} \forall g \\
& - \hat{A}b_g^{g \min} \leq v_g^{g \min} \leq \hat{A}b_g^{g \min} \forall g \\
& v_l^{t \max}, v_l^{t \min} \geq 0 \forall l \\
& v_g^{g \max}, v_g^{g \min} \geq 0 \forall g \\
& b_l^{t \max}, b_l^{t \min}, b_g^{g \max}, b_g^{g \min} \in \{1, 0\} \forall l, g \\
& t_N = 0
\end{aligned} \tag{7}$$

In (7), \hat{A} is a large enough number.

B. The worst-Nash equilibrium formulation

Each generating company is assumed as a Cournot player who is competing in a simultaneous-move game for having the

highest share from the market. The Nash solution concept is used to find the equilibrium of this game. To tackle the multiple Nash equilibria problem, the worst-Nash equilibrium is defined and mathematically modelled. The worst-Nash equilibrium is the one that has the maximum social cost to the society.

The mathematical formulation in (7) can be generalised as in (8).

$$\begin{aligned}
Max \pi_p(GCAP_p, GCAP_{-p}, F(GCAP_p, GCAP_{-p})) \\
s.t. \quad cs_p(GCAP_p, GCAP_{-p}, F(GCAP_p, GCAP_{-p})) \tag{8}
\end{aligned}$$

Where in (8), π_p is the profit function of GenCo p , CS_p represents the constraints of optimisation problem in (7), $GCAP_p = \{GCAP_g | \forall g \in GenCo_p\}$ is the quantity bidding set of GenCo p , $GCAP_{-p}$ is the quantity bidding set of other GenCos except p , and $F(GCAP_p, GCAP_{-p})$ represents other variables in optimisation problem (7).

The worst-Nash equilibrium formulation is given in (9).

$$\begin{aligned}
Min \sum_g \sum_n G_{g,n} MC_g QTY_g \\
s.t. \quad & \pi_p(GCAP_p^*, GCAP_{-p}^*, F(GCAP_p^*, GCAP_{-p}^*)) \geq \\
& \pi_p(GCAP'_p, GCAP_{-p}^*, F(GCAP'_p, GCAP_{-p}^*)) \forall p \\
& cs_p(GCAP_p^*, GCAP_{-p}^*, F(GCAP_p^*, GCAP_{-p}^*)) \forall p \\
& \pi_p(GCAP'_p, GCAP_{-p}^*, F(GCAP'_p, GCAP_{-p}^*)) \forall p
\end{aligned} \tag{9}$$

The worst-Nash equilibrium formulation is a mixed-integer nonlinear programming problem. The nonlinear term in (9) is the term $\sum_{g \in GenCo_p} v_g^{g \max} GCAP_g$ in the profit function π_p .

To linearise this term, we assume that the portfolio p can select its quantity bidding from a finite set. The number of this finite set can be any power of 2. Then we map each element of this set to a binary number. This can transform the nonlinear term in (9) to linear terms and a few constraints. The mathematics of this linearisation is detailed in (10).

Doing this, the mathematical structure in (9) is a mixed-integer linear programming problem. This linearized structure can efficiently locate the worst-Nash equilibrium of the Cournot game between Generating Companies in the market. The linearised Nash equilibrium solver in (9) can locate all Nash equilibria of the Cournot game between rival generating companies.

Also, it can be extended to include the Bertrand game and the price-quantity game.

$$\begin{aligned}
GCAP_g &= \frac{GMAX_g}{2^k} \sum_{s=1}^{k+1} 2^{s-1} b_s \Rightarrow \\
\sum_{g \notin GenCo_p} v_g^{g \max} GCAP_g &= \sum_{g \notin GenCo_p} v_g^{g \max} \frac{GMAX_g}{2^k} \sum_{s=1}^{k+1} 2^{s-1} b_s = \\
\sum_{g \notin GenCo_p} \frac{GMAX_g}{2^k} \sum_{s=1}^{k+1} 2^{s-1} \underbrace{v_g^{g \max} b_s}_y &= \\
\Rightarrow \sum_{g \notin GenCo_p} v_g^{g \max} GCAP_g &= \\
&\quad \sum_{g \notin GenCo_p} \frac{GMAX_g}{2^k} \sum_{s=1}^{k+1} 2^{s-1} \underbrace{v_g^{g \max} b_s}_y \\
&\quad 0 \leq y \leq \hat{A} b_s \\
&\quad 0 \leq y - v_g^{g \max} \leq \hat{A}(1 - b_s) \\
(10)
\end{aligned}$$

C. Transmission System Operator, TSO

This paper assumes a regulated Transmission System Operator, TSO. This regulated TSO maximises social welfare to the society in its assessment of transmission investment policies. The mathematical formulation of such a TSO is given in (11).

$$\begin{aligned}
\text{Min } & \sigma \sum_g \sum_n G_{g,n} MC_g GQTY_g + \sum_l IC_l TCAP_l \\
\text{s.t. } & \\
& \pi_p(GCAP_p^*, GCAP_{-p}^*, F(GCAP_p^*, GCAP_{-p}^*)) \geq \\
& \pi_p(GCAP'_p, GCAP_{-p}^*, F(GCAP'_p, GCAP_{-p}^*)) \forall p \\
& cs_p(GCAP_p^*, GCAP_{-p}^*, F(GCAP_p^*, GCAP_{-p}^*)) \forall p \\
& \pi_p(GCAP'_p, GCAP_{-p}^*, F(GCAP'_p, GCAP_{-p}^*)) \forall p \\
(11)
\end{aligned}$$

In (11), IC_l is the transmission investment cost for link l , and σ is the weighing factor which calculates the operating cost for all scenarios modelled in the transmission planning problem.

The derived mathematical structure in (11) models the market power cost in the assessment of transmission investment policies. This structure is a **closed-form analytical structure** and it **can be solved efficiently** using the available software packages.

Although the main focus of this paper is on mathematical derivation, a three-node example system is used to evaluate the operation of the derived mathematical structure. The structure (11) is solved using the CPLEX solver in the GAMS platform.

III. CASE STUDY

To show the effectiveness of the proposed approach, the model in (11) is applied to a three-node system, as illustrated in Fig. 1.

Transmission lines AB, AC, and BC connect buses A, B, and C. There are two competing generators labelled GenCo1 and GenCo2. The generating unit labelled as VoLL models the load shedding in the system. The characteristics of the generators, retailers, and the transmission network are set out in Table I, and Table II respectively.

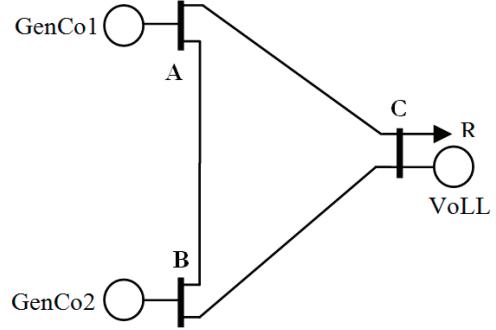


Fig. 1 The three-node example system

Table I Generators' data		
Generator	gmax (MW)	c (\$/MW)
GenCo1	1080	19.87
GenCo2	660	57.42
VoLL	10000	10000

Table II Transmission network data (power transfer distribution factor matrix and thermal limits)

Line#	GenCo1	GenCo2	VoLL	Limit(MW)
AB	0.3333	-0.3333	0	50
AC	0.6667	0.3333	0	10000
BC	0.3333	0.6667	0	10000

The total demand connected to node C is 1400 MW.

We assume a Cournot game in solving derived structure in (11) and also each generating unit is able to bid either half or full generation capacity.

The proposed model in (11) has been used to evaluate an upgrade of 100 MW for transmission line AB. The upgrade cost of this transmission line is assumed to be \$2,000,000.

The mathematical model in (11) approves upgrading of the link AB from 50 MW to 150 MW.

The worst-Nash equilibrium of GenCo1 and GenCo2 considering the 100MW augmentation of transmission system is found and presented in Table III.

As is clear from Table III, the additional 100 MW augmentation encourages GenCo2 to behave more competitively.

Table III The status of electricity market before and after 100 MW upgrading of link AB

GenCo	Quantity bidding (dispatched quantity)	
	Before	After
GenCo1	1080 (480) MW	540 (540) MW
GenCo2	330 (330) MW	660 (660) MW
VoLL	10000 (590) MW	10000 (200) MW
	11410 (1400) MW	11200 (1400) MW
worst Social Cost	\$ 5,928,441	\$ 2,048,627
Investment cost	0.0	\$ 2,000,000
Total TSO cost	\$ 5,928,441	\$ 4,048,627

In the augmented transmission system, GenCo2 offers 660MW of its capacity to the energy market. The transmission investment cost is \$2M and the social cost associated with the worst-Nash equilibrium of the market is \$2,048,627. This results in \$4,048,627 as the total TSO cost. Accordingly, adding a 100MW circuit between nodes A and B causes GenCo2 to offer 660MW of its capacity to the market - an improvement of 50% over its offered capacity before augmentation. The dispatch results before and after transmission system augmentation when the GenCos bid strategically are shown in Fig. 3 and Fig. 4.

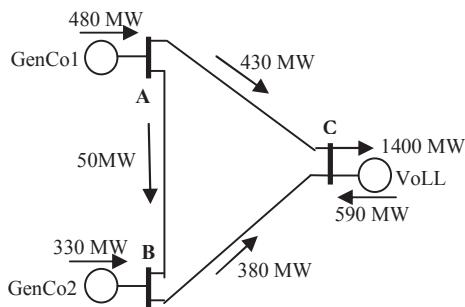


Fig. 3 The economic dispatch results of the three-node example system before transmission system augmentation, GenCos behave strategically

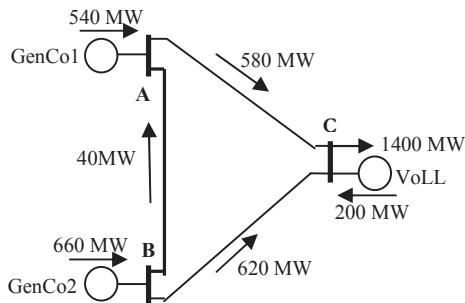


Fig. 4 The economic dispatch results of the three-node example system after transmission system augmentation, GenCos behave strategically (the augmented line is the thicker one)

The difference between the total TSO cost before and after augmentation is the total benefit of the transmission augmentation. The TSO cost before augmentation is \$ 5,928,441 and after augmentation is \$ 4,048,627. Then, the total benefit of 100 MW upgrade of link AB is \$ 1,879,814.

This example shows how the transmission upgrade approved by the proposed mathematical structure in (11) can force the electricity generating firms to behave more competitively.

This reduces the market power cost and increases the market competitiveness.

IV. CONCLUDING REMARKS

This paper derives a closed-form mathematical structure for modelling market power cost in the assessment of transmission investment policies.

The modelling is based on the concept of game theory in applied mathematics and the concept of social welfare in microeconomics. The developed mathematical structure is a mixed-integer linear programming problem.

In deriving the final mixed-integer linear programming problem, the following concepts are used:

- (a) The Karush-Kuhn-Tucker optimality conditions;
- (b) The disjunctive nature of the complementary slackness conditions;
- (c) The strong duality theorem; and
- (d) The mapping between binary values and the bidding quantities.

The derived mathematical framework is a **closed-form analytical structure** and it **can be solved efficiently** using the available software packages. Although the focus of this paper is on deriving the mathematical structure, but a three-node example system is used to show the operation of the structure. The numerical results show that the derived structure can evaluate the impact of additional transmission capacity on the strategic behaviours of the rival generating companies and consequently reducing the market power cost.

Using the proposed structure, the transmission investment policies can be used for improving both (a) the production efficiency and (b) the competitiveness in the electricity supply industry. However, more research is needed to make the proposed structure a practical tool for electricity market regulators and transmission system operators.

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